VLIDORT Notes 1: Inputs

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Abstract

In this Notes, I will give a brief introduction to the inputs for VLIDORT model, including the optical property inputs and linearized quantities. In particular, the detail derivation of the linearized quantities for calculating Jacobian of Stokes vector with respect to aerosol optical thickness, single scattering albedo, phase function, refractive index, and aerosol size parameter will be presented.

1 Basic optical property inputs

Generally, we consider the atmosphere a medium with Rayleigh scattering by air molecules, trace gas absorption, and scattering and absorption from aerosol particles. The basic optical property inputs including those information are given by[1]

$$\Delta = \alpha_{gas} + \sigma_{Ray} + \tau_{aer}, \tag{1}$$

$$\omega = \frac{\sigma_{aer} + \sigma_{Ray}}{\Delta}, \tag{2}$$

$$\mathbf{B}_l = \frac{\sigma_{aer}\mathbf{B}_{l,aer} + \sigma_{Ray}\mathbf{B}_{l,Ray}}{\sigma_{aer} + \sigma_{Ray}} \tag{3}$$

where σ_{Ray} is Rayleigh scattering optical depth, τ_{aer} and σ_{aer} are aerosol extinction and scattering optical depth, α_{gas} is gas absorption optical depth, $\mathbf{B}_l$ is the Greek matrix determined phase function Legendre expansion coefficients.

2 Linearized optical property inputs

In addition to Stokes vector $[I, Q, U, V]^T$, VLIDORT also has capability for the calculation of their Jacobian with respect to certain parameter ξ. The required normalized weighting functions for the linearized inputs, defined by

$$\phi_\xi = \frac{\xi}{\Delta}\frac{\partial \Delta}{\partial \xi}; \quad \varphi_\xi = \frac{\xi}{\omega}\frac{\partial \omega}{\partial \xi}; \quad \mathbf{\Psi}_{l,\xi} = \frac{\xi}{\mathbf{B}_l}\frac{\partial \mathbf{B}_l}{\partial \xi} \tag{4}$$

They could be established by differentiating the definitions in equations 1 – 3. Here I give the derived linearized inputs for retrieval Jacobian with respect to aerosol physical or optical parameters, such as optical thickness (τ_{aer}), aerosol single scattering albedo (ω_{aer}), and aerosol scattering phase function Legendre expansion coefficient ($\beta_{l,aer}$), size parameters, and refractive index.

2.1 A universal derivation: with respect to an arbitrary aerosol parameter x

Here we use x to represent an arbitrary aerosol parameter. The aerosol extinction and scattering optical thickness, single scattering alebdo, and phase function are functions of x. However, the gas absorption and Rayleigh scattering parameters are independent to x.

Then we drive equations (4) with $\xi = x$ by using equations 1 – 3.

$$\phi_x = \frac{x}{\Delta}\frac{\partial \tau_{aer}}{\partial x}, \tag{5}$$

$$\varphi_x = \frac{x}{\sigma_{aer}+\sigma_{Ray}}\frac{\partial \sigma_{aer}}{\partial x} - \phi_x, \tag{6}$$

$$\Psi_x^l = \frac{x}{\beta_l}\left(\frac{\sigma_{aer}}{\sigma_{aer}+\sigma_{Ray}}\frac{\partial \beta_{l,aer}}{\partial x} + \frac{\beta_{l,aer}-\beta_l}{\sigma_{aer}+\sigma_{Ray}}\frac{\partial \sigma_{aer}}{\partial x}\right) \tag{7}$$

Then, to finalize above 3 input parameters, we need to know 3 derivatives, $\partial\tau_{aer}/\partial x$, $\partial\sigma_{aer}/\partial x$, and $\partial\beta_{l,aer}/\partial x$. The following sections will tell how to calculate those 3 derivatives for given parameter x.

2.2 With respect to aerosol optical thickness: τ_{aer}

Let $x = \tau_{aer}$, we have

$$\frac{\partial \tau_{aer}}{\partial x} = 1, \tag{8}$$

$$\frac{\partial \sigma_{aer}}{\partial x} = \omega_{aer}, \tag{9}$$

$$\frac{\partial \beta_{l,aer}}{\partial x} = 0 \tag{10}$$

Replace those derivatives in equations 5 – 7, we have

$$\phi_{\tau_{aer}} = \frac{\tau_{aer}}{\Delta}, \tag{11}$$

$$\varphi_{\tau_{aer}} = \frac{\tau_{aer}}{\Delta}\left(\frac{\omega_{aer}}{\omega} - 1\right), \tag{12}$$

$$\Psi_{\tau_{aer}}^l = \begin{cases} \dfrac{\sigma_{aer}}{\sigma_{aer}+\sigma_{Ray}}\left(\dfrac{\beta_{l,aer}}{\beta_l} - 1\right) & \text{if } l \le 2 \\ \dfrac{\sigma_{Ray}}{\sigma_{aer}+\sigma_{Ray}} & \text{if } l \ge 3 \end{cases} \tag{13}$$

2.3 With respect to aerosol single scattering albedo: ω_{aer}

Let $x = \omega_{aer}$, we have

$$\begin{aligned} \frac{\partial \tau_{aer}}{\partial x} &= 0, & (14) \\ \frac{\partial \sigma_{aer}}{\partial x} &= \tau_{aer}, & (15) \\ \frac{\partial \beta_{l,aer}}{\partial x} &= 0 & (16) \end{aligned}$$

Replace those derivatives in equations 5 – 7, we have

$$\begin{aligned} \phi_{\omega_{aer}} &= 0, & (17) \\ \varphi_{\omega_{aer}} &= \frac{\sigma_{aer}}{\sigma_{aer} + \sigma_{Ray}}, & (18) \\ \Psi^l_{\omega_{aer}} &= \Psi^l_{\tau_{aer}} & (19) \end{aligned}$$

2.4 With respect to phase function: $\beta_{m,aer}$

$$\begin{aligned} \phi_{\beta_{m,aer}} &= 0, & (20) \\ \varphi_{\beta_{m,aer}} &= 0, & (21) \\ \Psi^l_{\beta_{m,aer}} &= \begin{cases} \dfrac{\sigma_{aer}\beta_{m,aer}}{\sigma_{aer}\beta_{m,aer} + \sigma_{Ray}\beta_{m,Ray}} & \text{if } m = l \\ 0 & \text{if } m \neq l \end{cases} & (22) \end{aligned}$$

2.5 With respect to aerosol refractive index: n_r

Both real and imaginary parts can be treated similarly. Here we use n_r to represent any of them two. τ_{aer} and σ_{aer} are functions of aerosol extinction and scattering efficiency (Q_{ext} and Q_{sca}), respectively.

$$\begin{aligned} \tau_{aer} &= \frac{3MQ_{ext}}{4\rho\, r_{eff}}, & (23) \\ \sigma_{aer} &= \frac{3MQ_{sca}}{4\rho\, r_{eff}} & (24) \end{aligned}$$

thus we obtain

$$\begin{aligned} \frac{\partial \tau_{aer}}{\partial n_r} &= \frac{\tau_{aer}}{Q_{ext}} \frac{\partial Q_{ext}}{\partial n_r}, & (25) \\ \frac{\partial \sigma_{aer}}{\partial n_r} &= \frac{\tau_{aer}}{Q_{ext}} \frac{\partial Q_{sca}}{\partial n_r} & (26) \end{aligned}$$

Now we only need $\partial Q_{ext}/\partial n_r$, $\partial Q_{sca}/\partial n_r$, and $\partial \beta_{l,aer}/\partial n_r$ for the linearized inputs calculation. Fortunately, we can acquire them from the the linearized Mie code.

2.6 With respect to aerosol size paramter: ν

Usually 2 or 3 parameters are used to quantify the size distribution of aerosol. For example, the lognormal size distribution can be established with effective radius and effective variance, or geometric radius and geometric variance. The linearized Mie code enable options for 8 type of size distribution, corresponding to 2 or 3 parameter for each of them. Here we use ν to represent certain one of them for the derivation of Linearized inputs for VLIDORT.

Note that, besides of (Q_{ext} and Q_{sca}), r_{eff} in equations 23 – 24 is also related to the size parameters. Then, we have

$$\frac{\partial \tau_{aer}}{\partial \nu} = \frac{\tau_{aer}}{Q_{ext}\, r_{eff}} \left(r_{eff} \frac{\partial Q_{ext}}{\partial \nu} - Q_{ext} \frac{\partial r_{eff}}{\partial \nu} \right), \tag{27}$$

$$\frac{\partial \sigma_{aer}}{\partial \nu} = \frac{\tau_{aer}}{Q_{ext}\, r_{eff}} \left(r_{eff} \frac{\partial Q_{sca}}{\partial \nu} - Q_{sca} \frac{\partial r_{eff}}{\partial \nu} \right) \tag{28}$$

The linearized Mie code is able to calculate the needed derivatives, $\partial Q_{ext}/\partial \nu$, $\partial Q_{sca}/\partial \nu$, $\partial \beta_{l,aer}/\partial \nu$, and $\partial r_{eff}/\partial \nu$.

References

[1] Robert Spurr, *User Guide: VLIDORT Version 2.3*. RT Solutions, Inc., MA, 2007.

Appendix A: Derivation of Linearized Inputs with Respect to an Arbitrary Aerosol Parameter

$$\begin{aligned}
\phi_x &= \frac{x}{\Delta}\frac{\partial \Delta}{\partial x} \\
&= \frac{x}{\Delta}\frac{\partial(\alpha_{gas} + \sigma_{Ray} + \tau_{aer})}{\partial x} \\
&= \frac{x}{\Delta}\frac{\partial \tau_{aer}}{\partial x},
\end{aligned} \tag{29}$$

$$\begin{aligned}
\varphi_x &= \frac{x}{\omega}\frac{\partial \omega}{\partial x} \\
&= \frac{x}{\omega}\frac{\partial[(\sigma_{aer} + \sigma_{Ray})/\Delta]}{\partial x} \\
&= \frac{x}{\omega}\frac{1}{\Delta^2}\left[\Delta\frac{\partial(\sigma_{aer} + \sigma_{Ray})}{\partial x} - (\sigma_{aer} + \sigma_{Ray})\frac{\partial \Delta}{\partial x}\right] \\
&= \frac{x}{\omega\Delta}\frac{\partial \sigma_{aer}}{\partial x} - (\sigma_{aer} + \sigma_{Ray})\frac{x}{\omega\Delta^2}\frac{\partial \tau_{aer}}{\partial x} \\
&= \frac{x}{(\sigma_{aer} + \sigma_{Ray})}\frac{\partial \sigma_{aer}}{\partial x} - \frac{x}{\Delta}\frac{\partial \tau_{aer}}{\partial x} \\
&= \frac{x}{(\sigma_{aer} + \sigma_{Ray})}\frac{\partial \sigma_{aer}}{\partial x} - \phi_x,
\end{aligned} \tag{30}$$

$$\begin{aligned}
\Psi_x^l &= \frac{x}{\beta_l}\frac{\partial \beta_l}{\partial x} \\
&= \frac{x}{\beta_l}\frac{\partial\left[(\sigma_{aer}\beta_{l,aer} + \sigma_{Ray}\beta_{l,Ray})/(\sigma_{aer} + \sigma_{Ray})\right]}{\partial x} \\
&= \frac{x}{\beta_l}\frac{1}{(\sigma_{aer} + \sigma_{Ray})^2}\left[(\sigma_{aer} + \sigma_{Ray})\frac{\partial(\sigma_{aer}\beta_{l,aer})}{\partial x}\right. \\
&\quad \left. -(\sigma_{aer}\beta_{l,aer} + \sigma_{Ray}\beta_{l,Ray})\frac{\partial \sigma_{aer}}{\partial x}\right] \\
&= \frac{x}{\beta_l}\frac{1}{\sigma_{aer} + \sigma_{Ray}}\left[\frac{\partial(\sigma_{aer}\beta_{l,aer})}{\partial x} - \beta_l\frac{\partial \sigma_{aer}}{\partial x}\right] \\
&= \frac{x}{\beta_l}\frac{1}{\sigma_{aer} + \sigma_{Ray}}\left[\sigma_{aer}\frac{\partial \beta_{l,aer}}{\partial x} + \beta_{l,aer}\frac{\partial \sigma_{aer}}{\partial x} - \beta_l\frac{\partial \sigma_{aer}}{\partial x}\right] \\
&= \frac{x}{\beta_l}\frac{1}{\sigma_{aer} + \sigma_{Ray}}\left[\sigma_{aer}\frac{\partial \beta_{l,aer}}{\partial x} + (\beta_{l,aer} - \beta_l)\frac{\partial \sigma_{aer}}{\partial x}\right]
\end{aligned} \tag{31}$$