VLIDORT Notes 2: Output Jacobians

Xiaoguang Xu
Dept. of Geosciences,
University of Nebraska-Lincoln
$xxu@huskers.unl.edu$

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Abstract

In this Notes, I will give a brief introduction to the outputs for VLIDORT model, including the radiation outputs and their Jacobians. In particular, definition of Jacobian, relationship between atmospheric column Jacobian and profile Jacobian will be presented.

1 Basic radiation outputs

Generally, radiance and polarization are what we are interested, which usually expressed by the Stokes vector, $[I, Q, U, V]^T$. VLIDORT has capacity to calculate the Stokes vector on different atmospheric level for certain sun-view geometry and for both down-and-upwelling. One can specify the input variable "N_USER_LEVELS" and "USER_LEVELS" to determine the Stokes vector on which atmospheric level will output. For instance, N_USER_LEVELS=1, and USER_LEVELS(1) = 0.0 will make the model output the variable at top-of-atmosphere.

2 Jacobian outputs

2.1 The definition of Jacobian

In addition to Stokes vector $\mathrm{S} = [I, Q, U, V]^T$, VLIDORT also has capability for the calculation of their Jacobian with respect to certain parameter ξ. The required normalized weighting functions for the linearized inputs, defined by

$$\mathrm{S}_\xi = \xi \frac{\partial \mathrm{S}}{\partial \xi} \tag{1}$$

ξ could be optical or microphysical parameters for trace gas, Rayleigh scattering, or aerosols. The inputs to VLIDORT for calculating the Jacobian has been discussed in VLIDORT NOTES 1.

3 Jacobian of Degree of Polarization

Given the Jacobian of Stokes vector with respect to certain parameter, the corresponding Jacobian of degree of polarization, P with respect to that parameter can be derived. Let P be the degree of linear polarization

$$P = \frac{\sqrt{Q^2 + U^2}}{I} \tag{2}$$

We can use the chain rule to obtain

$$\begin{aligned} \xi \frac{\partial P}{\partial \xi} &= \xi \frac{\partial P}{\partial I} \frac{\partial I}{\partial \xi} + \xi \frac{\partial P}{\partial Q} \frac{\partial Q}{\partial \xi} + \xi \frac{\partial P}{\partial U} \frac{\partial U}{\partial \xi} \\ &= \xi \frac{\partial I}{\partial \xi} (-\frac{1}{I^2} \sqrt{Q^2 + U^2}) + \frac{1}{I\sqrt{Q^2 + U^2}} (\xi \frac{\partial Q}{\partial \xi} Q + \xi \frac{\partial U}{\partial \xi} U) \\ &= \xi \frac{\partial I}{\partial \xi} (-\frac{P}{I}) + \frac{1}{I\sqrt{Q^2 + U^2}} (\xi \frac{\partial Q}{\partial \xi} Q + \xi \frac{\partial U}{\partial \xi} U) \end{aligned} \tag{3}$$

References

[1] Robert Spurr, *User Guide: VLIDORT Version 2.3*. RT Solutions, Inc., MA, 2007.