UNL-VRTM Notes 4
Aerosol Vertical Profiles

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Abstract

This note discusses the aerosol vertical profiles, including uniform loading, exponential loading, and Gaussian loading.

1 Exponential Loading

Usually, the exponentially-decreasing profile relates to the scale height H. Considering an ideal atmosphere from the TOA at infinite height to any altitude z, the optical depth is defined by

$$\tau(z) = \tau_0 e^{-\frac{z}{H}} \tag{1}$$

Therefore, the values of τ at surface and TOA are respectively,

$$\tau(z = 0) = \tau_0 \tag{2}$$
$$\tau(z = +\infty) = 0 \tag{3}$$

Aerosol usually present within certain range of atmospheric profile. Let the upper and lower level height of this range be z_1 and z_2, respectively ($z_1 > z_2$), we can then write

$$\tau(z) = b\left(e^{-\frac{z}{H}} - e^{-\frac{z_1}{H}}\right), \tag{4}$$

where b is a normalization constant related to the columnar optical depth τ_0,

$$b = \frac{\tau_0}{e^{-\frac{z_2}{H}} - e^{-\frac{z_1}{H}}} \tag{5}$$

The aerosol optical thickness of any atmospheric layer (with aerosol present) is then $\tau_n = \int_{z_{n-1}}^{z_n} \tau(z)\mathrm{d}z$, which gives

$$\tau_n = b\left(e^{-\frac{z_n}{H}} - e^{-\frac{z_{n-1}}{H}}\right), \tag{6}$$

where z_n and z_{n-1} are the lower and upper boundary height of this layer, respectively.

Differentiation of above equation with respect to τ_0 and H yields

$$\frac{\partial \tau_n}{\partial \tau_0} = \frac{\tau_n}{\tau_0} \tag{7}$$

$$\frac{\partial \tau_n}{\partial H} = \frac{\partial \tau_n}{\partial k}\frac{\partial k}{\partial H} \tag{8}$$

Here we define $k = \frac{1}{H}$ for the convenience of our derivation, $\frac{\partial k}{\partial H} = -\frac{1}{H^2}$. And $\frac{\partial \tau_n}{\partial k}$ can be obtained by

$$\begin{aligned}\frac{\partial \tau_n}{\partial k} &= \frac{\partial b}{\partial k}\frac{\tau_n}{b} + b\frac{\partial\left(e^{-kz_n} - e^{-kz_{n-1}}\right)}{\partial k} \\ &= \frac{\partial b}{\partial k}\frac{\tau_n}{b} + b\left(-z_n e^{-kz_n} + z_{n-1}e^{-kz_{n-1}}\right).\end{aligned} \tag{9}$$

$\frac{\partial b}{\partial k}$ is a constant:

$$\begin{aligned}\frac{\partial b}{\partial k} &= -\frac{\tau_0\left(-z_2 e^{-kz_2} + z_1 e^{-kz_1}\right)}{\left(e^{-kz_2} - e^{-kz_1}\right)^2} \\ &= -\frac{b^2}{\tau_0}\left(-z_2 e^{-kz_2} + z_1 e^{-kz_1}\right)\end{aligned} \tag{10}$$